

Single-step genomic selection: accommodating several key issues

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Overview

- **Genomic selection**

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- **Motivating Example**

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- **Complexities**

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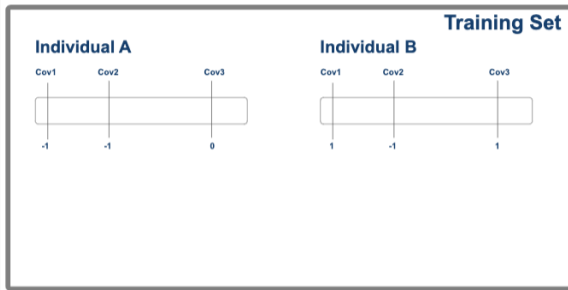
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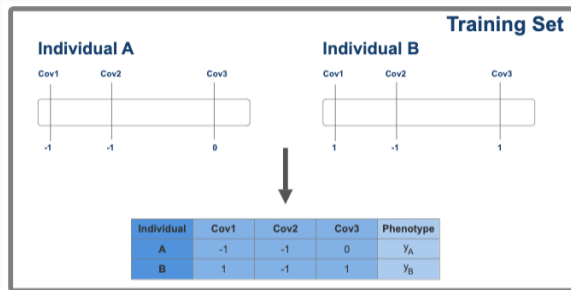
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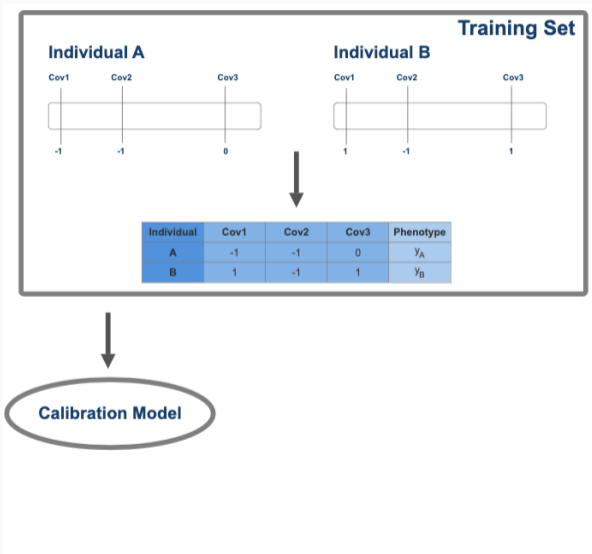
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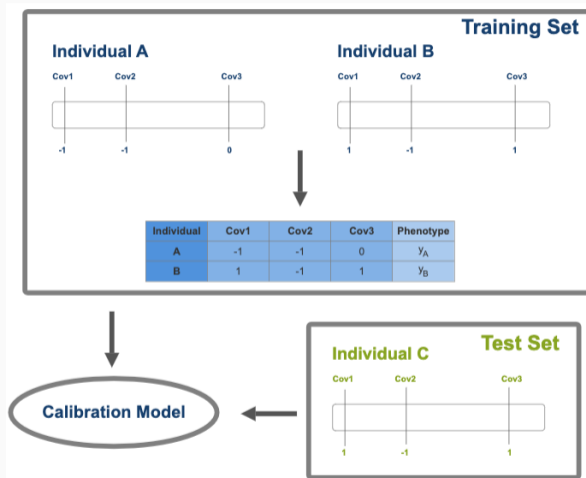
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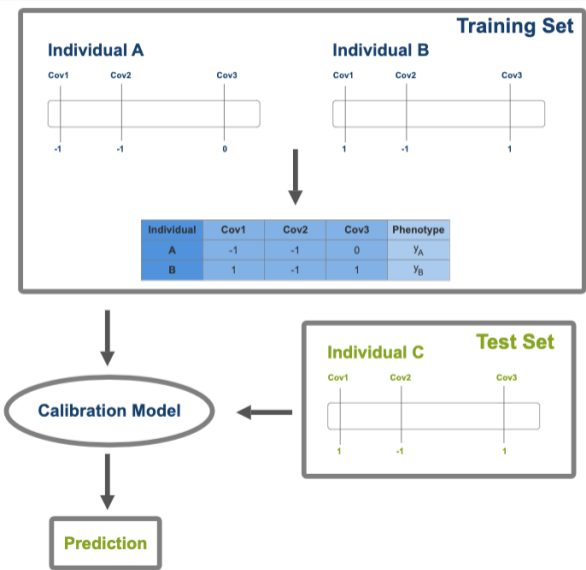
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- For the additive VE effects \mathbf{u}_a , covariance between varieties is also modelled via a known relationship matrix denoted \mathbf{G}

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- The matrix formed via ancestral records is denoted **A** and known as the numerator relationship matrix (NRM) (Oakey et al. [2007])

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- The final model fit included $k_a = 2$ additive and $k_e = 1$ non-additive factors

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- Several strategies exist to overcome this complexity however the method we have used is a blending of the **K** and **A** relationship matrices

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- The added parameter λ controls the weighting from the two sources of relatedness, with genomic information dominating when $\lambda \approx 1$ and pedigree information dominating when $\lambda \approx 0$

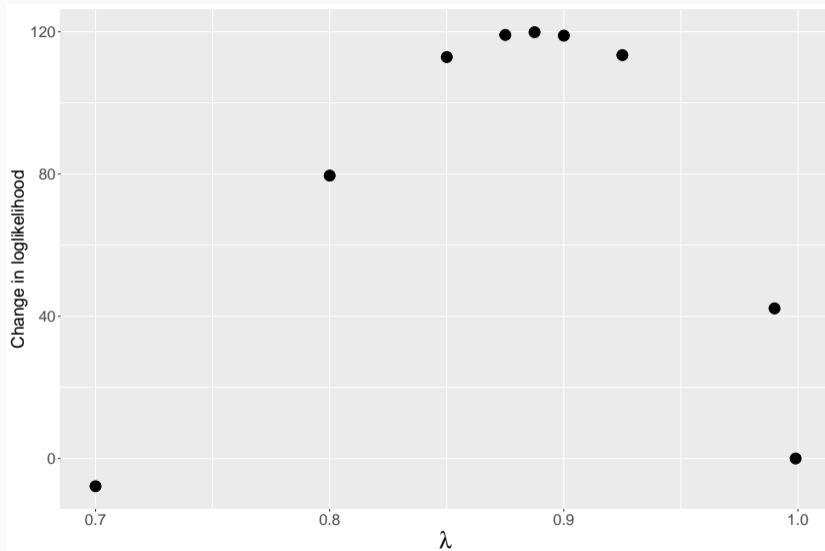
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- However this can be shown graphically through profiling on the parameter of interest (λ) and assessing the residual log-likelihood under a range of values for this parameter

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- However many complexities arise in implementation, a few of which have been explored here, specifically singularities in the GRM (\mathbf{K})
- Combining both ancestral and genomic relationship matrices both overcomes this issue
- Finally DWReml (Butler, pers comm) can handle the high computational demand when fitting complex single-step factor models which include genetic relatedness such as the motivating example

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